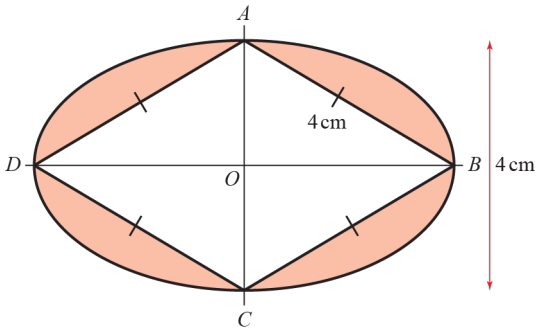
1 Figure 1 shows a logo comprised of a rhombus surrounded by two arcs. Arc BAD has centre C and arc BCD has centre A. Some of the dimensions of the logo are shown in the diagram.

Figure 1



Prove that the shaded area of the logo is $\frac{2}{3}\left(16\pi - 24\sqrt{3}\right)$ **(8 marks)**

2 **a** When θ is small, show that the equation $\frac{1+\sin\theta+\tan 2\theta}{2\cos 3\theta - 1}$ can be written as

$$\frac{1}{1-3\theta}$$ **(4 marks)**

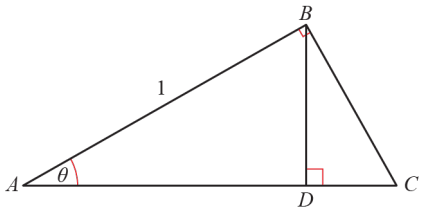
b Hence write down the value of $\frac{1+\sin\theta+\tan 2\theta}{2\cos 3\theta - 1}$ when θ is small. **(1 mark)**

3 **a** Prove that $\frac{\tan x - \sec x}{1-\sin x} \equiv -\sec x, \quad x \neq (2n+1)\frac{\pi}{2}$ **(3 marks)**

b Hence solve, in the interval $0 \leq x \leq 2\pi$, the equation $\frac{\tan x - \sec x}{1-\sin x} = \sqrt{2}$ **(3 marks)**

4 Figure 2 shows the right-angled triangles ΔABC, ΔABD and ΔBCD, with $AB = 1$ and $\angle BAD = \theta$.

Figure 2



Prove that $1 + \tan^2\theta = \sec^2\theta$ **(8 marks)**

5 Solve $6\sin(\theta + 60) = 8\sqrt{3}\cos\theta$ in the range $0 \leq \theta \leq 360°$. Round your answer to 1 decimal place. **(4 marks)**

6 **a** Prove that $(\sin 3\theta + \cos 3\theta)^2 \equiv 1 + \sin 6\theta$ **(3 marks)**

b Use the result to solve, for $0 \leq \theta \leq \frac{\pi}{2}$, the equation $(\sin 3\theta + \cos 3\theta) = \sqrt{\frac{2+\sqrt{2}}{2}}$

Give your answer in terms of π. Check for extraneous solutions. **(4 marks)**

7 **a** Express $5\cos\theta - 8\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \pi$

Write R in surd form and give the value of α correct to 4 decimal places. **(4 marks)**

The temperature of a kiln, $T°C$, used to make pottery can be modelled by the equation $T = 1100 + 5\cos\left(\frac{x}{3}\right) - 8\sin\left(\frac{x}{3}\right)$, $0 \leq x \leq 72$ where x is the time in hours since the pottery was placed in the kiln.

b Calculate the maximum value of T predicted by this model and the value of x, to 2 decimal places, when this maximum first occurs. **(4 marks)**

c Calculate the times during the first 24 hours when the temperature is predicted, by this model, to be exactly 1097 °C. **(4 marks)**