

Write your name here

Surname	Other names
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Pearson

Edexcel GCE

Centre Number

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Candidate Number

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A level Mathematics

Practice Paper

Pure Mathematics - Trigonometry (part 2)

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You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

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Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
- There are 10 questions in this question paper. The total mark for this paper is 100.
- The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.
- Calculators must not be used for questions marked with a * sign.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1. (a) Express $2 \cos \theta - \sin \theta$ in the form $R \cos (\theta + \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < 90^\circ$. Give the exact value of R and give the value of α to 2 decimal places.

(3)

- (b) Hence solve, for $0 \leq \theta < 360^\circ$,

$$\frac{2}{2 \cos \theta - \sin \theta - 1} = 15.$$

Give your answers to one decimal place.

(5)

- (c) Use your solutions to parts (a) and (b) to deduce the smallest positive value of θ for which

$$\frac{2}{2 \cos \theta + \sin \theta - 1} = 15.$$

Give your answer to one decimal place.

(2)

(Total 10 marks)

2. $g(\theta) = 4 \cos 2\theta + 2 \sin 2\theta$.

Given that $g(\theta) = R \cos (2\theta - \alpha)$, where $R > 0$ and $0 < \alpha < 90^\circ$,

- (a) find the exact value of R and the value of α to 2 decimal places.

(3)

- (b) Hence solve, for $-90^\circ < \theta < 90^\circ$,

$$4 \cos 2\theta + 2 \sin 2\theta = 1,$$

giving your answers to one decimal place.

(5)

Given that k is a constant and the equation $g(\theta) = k$ has no solutions,

- (c) state the range of possible values of k .

(2)

(Total 10 marks)

3. Given that

$$2 \cos (x + 50)^{\circ} = \sin (x + 40)^{\circ}.$$

(a) Show, without using a calculator, that

$$\tan x^{\circ} = \frac{1}{3} \tan 40^{\circ}.$$

(4)

(b) Hence solve, for $0 \leq \theta < 360$,

$$2 \cos (2\theta + 50)^{\circ} = \sin (2\theta + 40)^{\circ},$$

giving your answers to 1 decimal place.

(4)

(Total 8 marks)

4. Find all the solutions of

$$2 \cos 2\theta = 1 - 2 \sin \theta$$

in the interval $0 \leq \theta < 360^{\circ}$.

(Total 6 marks)

5. (a) Write $5 \cos \theta - 2 \sin \theta$ in the form $R \cos (\theta + \alpha)$, where R and α are constants,

$$R > 0 \text{ and } 0 \leq \alpha < \frac{\pi}{2}$$

Give the exact value of R and give the value of α in radians to 3 decimal places.

(3)

- (b) Show that the equation

$$5 \cot 2x - 3 \operatorname{cosec} 2x = 2$$

can be rewritten in the form

$$5 \cos 2x - 2 \sin 2x = c$$

where c is a positive constant to be determined.

(2)

- (c) Hence or otherwise, solve, for $0 \leq x < \pi$,

$$5 \cot 2x - 3 \operatorname{cosec} 2x = 2$$

giving your answers to 2 decimal places.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(4)

(Total 9 marks)

6. (a) Express $6 \cos \theta + 8 \sin \theta$ in the form $R \cos (\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the value of α to 3 decimal places.

(4)

- (b)
$$p(\theta) = \frac{4}{12 + 6 \cos \theta + 8 \sin \theta}, \quad 0 \leq \theta \leq 2\pi.$$

Calculate

- (i) the maximum value of $p(\theta)$,

- (ii) the value of θ at which the maximum occurs.

(4)

(Total 8 marks)

7. (i) Without using a calculator, find the exact value of

$$(\sin 22.5^\circ + \cos 22.5^\circ)^2.$$

You must show each stage of your working.

(5)

- (ii) (a) Show that $\cos 2\theta + \sin \theta = 1$ may be written in the form

$$k \sin^2 \theta - \sin \theta = 0, \text{ stating the value of } k.$$

(2)

- (b) Hence solve, for $0 \leq \theta < 360^\circ$, the equation

$$\cos 2\theta + \sin \theta = 1.$$

(4)

(Total 11 marks)

8.

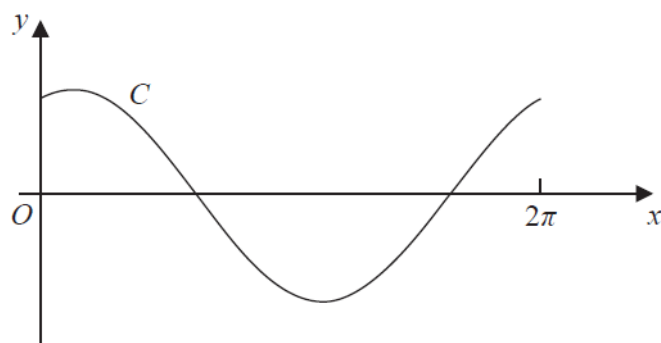


Figure 1

Figure 1 shows the curve C , with equation $y = 6 \cos x + 2.5 \sin x$ for $0 \leq x \leq 2\pi$.

- (a) Express $6 \cos x + 2.5 \sin x$ in the form $R \cos(x - \alpha)$, where R and α are constants with $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give your value of α to 3 decimal places. (3)
- (b) Find the coordinates of the points on the graph where the curve C crosses the coordinate axes. (3)

A student records the number of hours of daylight each Sunday throughout the year. She starts on the last Sunday in May with a recording of 18 hours, and continues until her final recording 52 weeks later.

She models her results with the continuous function given by

$$H = 12 + 6 \cos\left(\frac{2\pi t}{52}\right) + 2.5 \sin\left(\frac{2\pi t}{52}\right), \quad 0 \leq t \leq 52$$

where H is the number of hours of daylight and t is the number of weeks since her first recording.

Use this function to find

- (c) the maximum and minimum values of H predicted by the model, (3)
- (d) the values for t when $H = 16$, giving your answers to the nearest whole number.

[You must show your working. Answers based entirely on graphical or numerical methods are not acceptable.] (6)

(Total 15 marks)

9.

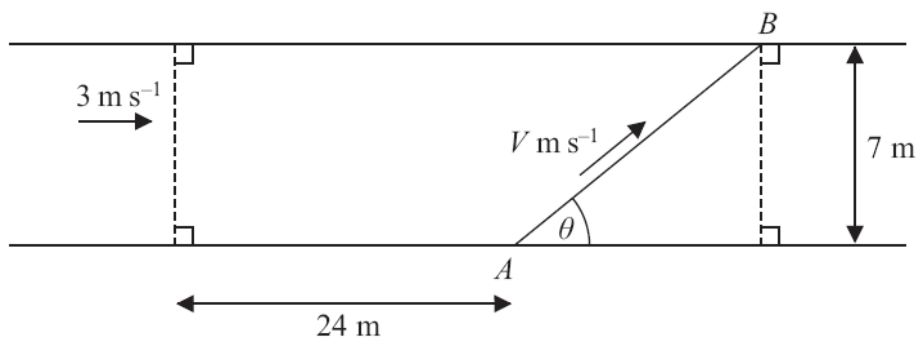


Figure 2

Kate crosses a road, of constant width 7 m, in order to take a photograph of a marathon runner, John, approaching at 3 m s^{-1} .

Kate is 24 m ahead of John when she starts to cross the road from the fixed point A . John passes her as she reaches the other side of the road at a variable point B , as shown in Figure 2.

Kate's speed is $V \text{ m s}^{-1}$ and she moves in a straight line, which makes an angle θ , $0 < \theta < 150^\circ$, with the edge of the road, as shown in Figure 2.

You may assume that V is given by the formula

$$V = \frac{21}{24 \sin \theta + 7 \cos \theta}, \quad 0 < \theta < 150^\circ$$

- (a) Express $24 \sin \theta + 7 \cos \theta$ in the form $R \cos(\theta - \alpha)$, where R and α are constants and where $R > 0$ and $0 < \alpha < 90^\circ$, giving the value of α to 2 decimal places. (3)

Given that θ varies,

- (b) find the minimum value of V . (2)

Given that Kate's speed has the value found in part (b),

- (c) find the distance AB . (3)

Given instead that Kate's speed is 1.68 m s^{-1} ,

- (d) find the two possible values of the angle θ , given that $0 < \theta < 150^\circ$. (6)

(Total 14 marks)

10. (a) Express $2 \sin \theta - 4 \cos \theta$ in the form $R \sin(\theta - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the value of α to 3 decimal places.

(3)

$$H(\theta) = 4 + 5(2\sin 3\theta - 4\cos 3\theta)^2$$

Find

- (b) (i) the maximum value of $H(\theta)$,
(ii) the smallest value of θ , for $0 \leq \theta \leq \pi$, at which this maximum value occurs.

(3)

Find

- (c) (i) the minimum value of $H(\theta)$,
(ii) the largest value of θ , for $0 \leq \theta \leq \pi$, at which this minimum value occurs.

(3)

(Total 9 marks)

TOTAL FOR PAPER: 100 MARKS