

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
1	Use Pythagoras' theorem to show that the length of $OB = 2\sqrt{3}$ or $OD = 2\sqrt{3}$ or states $BD = 4\sqrt{3}$	M1	2.2a	6th Solve problems involving arc length and sector area in context.
	Makes an attempt to find $\angle DAB$ or $\angle DCB$ . For example, $\cos \angle DAO = \frac{2}{4}$ is seen.	M1	2.2a	
	Correctly states that $\angle DAB = \frac{2\pi}{3}$ or $\angle DCB = \frac{2\pi}{3}$	A1	1.1b	
	Makes an attempt to find the area of the sector with a radius of 4 and a subtended angle of $\frac{2\pi}{3}$ For example, $A = \frac{1}{2} \times 4^2 \times \frac{2\pi}{3}$ is shown.	M1	2.2a	
	Correctly states that the area of the sector is $\frac{16\pi}{3}$	A1	1.1b	
	Recognises the need to subtract the sector area from the area of the rhombus in an attempt to find the shaded area. For example, $\frac{16\pi}{3} - 8\sqrt{3}$ is seen.	M1	3.2a	
	Recognises that to find the total shaded area this number will need to be multiplied by 2. For example, $2 \times \left( \frac{16\pi}{3} - 8\sqrt{3} \right)$	M1	3.2a	
	Using clear algebra, correctly manipulates the expression and gives a clear final answer of $\frac{2}{3}(16\pi - 24\sqrt{3})$	A1	1.1b	
<b>(8 marks)</b>				
<b>Notes</b>				

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<b>2a</b>	Shows that $2\cos 3\theta \approx 2\left(1 - \frac{9\theta^2}{2}\right) = 2 - 9\theta^2$	<b>M1</b>	2.1	6th Understand small-angle approximations for sin, cos and tan (angle in radians).	
	Shows that $2\cos 3\theta - 1 \approx 1 - 9\theta^2 = (1 - 3\theta)(1 + 3\theta)$	<b>M1</b>	1.1b		
	Shows $1 + \sin \theta + \tan 2\theta = 1 + \theta + 2\theta = 1 + 3\theta$	<b>M1</b>	2.1		
	Recognises that $\frac{1 + \sin \theta + \tan 2\theta}{2\cos 3\theta - 1} \approx \frac{1 + 3\theta}{(1 - 3\theta)(1 + 3\theta)} = \frac{1}{1 - 3\theta}$	<b>A1</b>	1.1b		
		<b>(4)</b>			
<b>2b</b>	When $\theta$ is small, $\frac{1}{1 - 3\theta} \approx 1$	<b>A1</b>	1.1b	7th Use small-angle approximations to solve problems.	
		<b>(1)</b>			
<b>(5 marks)</b>					
<b>Notes</b>					

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor	
3a	Writes $\tan x$ and $\sec x$ in terms of $\sin x$ and $\cos x$ . For example, $\frac{\tan x - \sec x}{1 - \sin x} = \frac{\left(\frac{\sin x}{\cos x} - \frac{1}{\cos x}\right)}{\left(\frac{1 - \sin x}{1}\right)}$	M1	2.1	5th Understand the functions sec, cosec and cot.	
	Manipulates the expression to find $\left(\frac{\sin x - 1}{\cos x}\right) \times \left(\frac{1}{1 - \sin x}\right)$	M1	1.1b		
	Simplifies to find $-\frac{1}{\cos x} = -\sec x$	A1	1.1b		
		(3)			
3b	States that $-\sec x = \sqrt{2}$ or $\sec x = -\sqrt{2}$	B1	2.2a	6th Use the functions sec, cosec and cot to solve simple trigonometric problems.	
	Writes that $\cos x = -\frac{1}{\sqrt{2}}$ or $x = \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$	M1	1.1b		
	Finds $x = \frac{3\pi}{4}, \frac{5\pi}{4}$	A1	1.1b		
		(3)			
(6 marks)					
Notes					

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4	States that $\sin \theta = \frac{BD}{1}$ and concludes that $BD = \sin \theta$	M1	3.1	6th Prove $\sec^2 x = 1 + \tan^2 x$ and $\operatorname{cosec}^2 x = 1 + \cot^2 x$ .
	States that $\cos \theta = \frac{AD}{1}$ and concludes that $AD = \cos \theta$	M1	3.1	
	States that $\angle DBC = \theta$	M1	2.2a	
	States that $\tan \theta = \frac{DC}{\sin \theta}$ and concludes that $DC = \frac{\sin^2 \theta}{\cos \theta}$ oe.	M1	3.1	
	States that $\cos \theta = \frac{\sin \theta}{BC}$ and concludes that $BC = \tan \theta$ oe.	M1	3.1	
	Recognises the need to use Pythagoras' theorem. For example, $AB^2 + BC^2 = AC^2$	M1	2.2a	
	Makes substitutions and begins to manipulate the equation: $1 + \tan^2 \theta = \left( \frac{\cos \theta}{1} + \frac{\sin^2 \theta}{\cos \theta} \right)^2$ $1 + \tan^2 \theta = \left( \frac{\cos^2 \theta + \sin^2 \theta}{\cos \theta} \right)^2$	M1	1.1b	
	Uses a clear algebraic progression to arrive at the final answer: $1 + \tan^2 \theta = \left( \frac{1}{\cos \theta} \right)^2$ $1 + \tan^2 \theta = \sec^2 \theta$	A1	1.1b	
				(8 marks)
	Notes			

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5	Uses the double-angle formulae to write: $6\sin\theta\cos 60 + 6\cos\theta\sin 60 = 8\sqrt{3}\cos\theta$	M1	2.2a	6th Use the double-angle formulae for sin, cos and tan.	
	Uses the fact that $\cos 60 = \frac{1}{2}$ and $\sin 60 = \frac{\sqrt{3}}{2}$ to write: $3\sin\theta + 3\sqrt{3}\cos\theta = 8\sqrt{3}\cos\theta$	M1	1.1b		
	Simplifies this expression to $\tan\theta = \frac{5\sqrt{3}}{3}$	M1	1.1b		
	Correctly solves to find $\theta = 70.9^\circ, 250.9^\circ$	A1	1.1b		
(4 marks)					
Notes					

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6a	<p>Writes <math>(\sin 3\theta + \cos 3\theta)^2 \equiv (\sin 3\theta + \cos 3\theta)(\sin 3\theta + \cos 3\theta)</math>  <math>\equiv \sin^2 3\theta + 2 \sin 3\theta \cos 3\theta + \cos^2 3\theta</math></p> <p>Uses <math>\sin^2 3\theta + \cos^2 3\theta \equiv 1</math> and <math>2 \sin 3\theta \cos 3\theta \equiv \sin 6\theta</math> to write:  <math>(\sin 3\theta + \cos 3\theta)^2 \equiv 1 + \sin 6\theta</math></p> <p>Award one mark for each correct use of a trigonometric identity.</p>	M1	1.1b	<p>7th  Use addition formulae and/or double-angle formulae to solve equations.</p>
		(3)		
6b	<p>States that:</p> $1 + \sin 6\theta = \frac{2 + \sqrt{2}}{2}$ <p>Simplifies this to write:</p> $\sin 6\theta = \frac{\sqrt{2}}{2}$ <p>Correctly finds <math>6\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}</math></p> <p>Additional answers might be seen, but not necessary in order to award the mark.</p> <p>States <math>\theta = \frac{\pi}{24}, \frac{3\pi}{24}</math></p> <p>Note that <math>\theta \neq \frac{9\pi}{24}, \frac{11\pi}{24}</math>. For these values <math>3\theta</math> lies in the third quadrant, therefore <math>\sin 3\theta</math> and <math>\cos 3\theta</math> are both negative and cannot be equal to a positive surd.</p>	B1	2.2a	<p>7th  Use addition formulae and/or double-angle formulae to solve equations.</p>
		(4)		
				(7 marks)
6b	<p>Award all 4 marks if correct final answer is seen, even if some of the <math>6\theta</math> angles are missing in the preceding step.</p>			

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
7a	<p>States:</p> $R\cos(\theta + \alpha) \equiv R\cos\theta\cos\alpha - R\sin\theta\sin\alpha$ <p>Or:</p> $5\cos\theta - 8\sin\theta \equiv R\cos\theta\cos\alpha - R\sin\theta\sin\alpha$ <p>Deduces that:</p> $5 = R\cos\alpha \quad 8 = R\sin\alpha$ <p>States that <math>R = \sqrt{89}</math></p> <p>Use of <math>\sin^2\theta + \cos^2\theta = 1</math> might be seen, but is not necessary to award the mark.</p> <p>Finds that <math>\alpha = 1.0122</math></p> <p><math>\tan\alpha = \frac{8}{5}</math> might be seen, but is not necessary to award the mark.</p>	M1	1.1b	6th Understand how to use identities to rewrite $a\cos x + b\sin x$ .
7b	<p>Uses the maths from part a to deduce that</p> $T_{\max} = 1100 + \sqrt{89} = 1109.43^\circ C$	A1	3.4	7th Solve problems involving $a\cos x + b\sin x$ .
	<p>Recognises that the maximum temperature occurs when</p> $\cos\left(\frac{x}{3} + 1.0122\right) = 1$	M1	3.4	
	<p>Solves this equation to find <math>\frac{x}{3} = 2\pi - 1.0122</math></p>	M1	1.1b	
	<p>Finds <math>x = 15.81</math> hours</p>	A1	1.1b	
		(4)		

7c	Deduces that $1097 = 1100 + \sqrt{89} \cos\left(\frac{x}{3} + 1.0122\right)$	M1	3.4	8th Use trigonometric functions and identities to solve problems in a range of unfamiliar contexts.	
	Begins to solve the equation. For example, $\cos\left(\frac{x}{3} + 1.0122\right) = -\frac{3}{\sqrt{89}}$ is seen.	M1	1.1b		
	States that $\frac{x}{3} + 1.0122 = 1.8944, 2\pi - 1.8944, 2\pi + 1.8944$ Further values may be seen, but are not necessary in order to award the mark.	M1	1.1b		
	Finds that $x = 2.65$ hours, $10.13$ hours, $21.50$ hours	A1	1.1b		
		(4)			
(12 marks)					
Notes					