1 It is suggested that the sequence $a_k = 2^k + 1, k$ Ö 1 produces only prime numbers.

a Show that a_1, a_2 and a_4 produce prime numbers. **(2 marks)**

b Prove by counter example that the sequence does not always produce a prime number. **(2 marks)**

2 Prove by exhaustion that $1+2+3+...+n \equiv \frac{n(n+1)}{2}$ for positive integers from 1 to 6 inclusive. **(3 marks)**

3 Use proof by contradiction to prove the statement: 'The product of two odd numbers is odd.' **(5 marks)**

4 Prove by contradiction that if n is odd, $n^3 + 1$ is even. **(5 marks)**

5 Use proof by contradiction to show that there exist no integers a and b for which $25a + 15b = 1$. **(4 marks)**

6 Use proof by contradiction to show that there is no greatest positive rational number. **(4 marks)**

7 Use proof by contradiction to show that, given a rational number a and an irrational number b, $a - b$ is irrational. **(4 marks)**

8 Use proof by contradiction to show that there are no positive integer solutions to the statement $x^2 - y^2 = 1$. **(5 marks)**

9 **a** Use proof by contradiction to show that if n^2 is an even integer then n is also an even integer. **(4 marks)**

b Prove that $\sqrt{2}$ is irrational. **(6 marks)**

10 Prove by contradiction that there are infinitely many prime numbers. **(6 marks)**