SelexIQ Education – A Level Mathematics
Chapter 2: Functions Worksheet

Instructions

- Read each question carefully before attempting your solution.
- Show all relevant working for full marks.
- You may use a scientific calculator unless otherwise stated.
- Answer all questions in the spaces provided.

Student Name: _____

Date: _____

Questions

Question 1

The functions f and g are defined by

$$f(x) = 7 - 2x^2 \qquad x \in \mathbb{R}$$

$$g(x) = \frac{3x}{5x-1} \qquad x \in \mathbb{R} \qquad x \neq \frac{1}{5}$$

(a) State the range of f

(1)

(b) Find $gf(1.8)$

(2)

(c) Find $g^{-1}(x)$

(2)

Question 2

The function f is defined by

$$f(x) = \frac{8x + 5}{2x + 3} \qquad x > -\frac{3}{2}$$

(a) Find $f^{-1}\left(\frac{3}{2}\right)$

(2)

(b) Show that

$$f(x) = A + \frac{B}{2x + 3}$$

where A and B are constants to be found.

(2)

The function g is defined by

$$g(x) = 16 - x^2 \qquad 0 \leqslant x \leqslant 4$$

(c) State the range of g^{-1}

(1)

(d) Find the range of $f g^{-1}$

(3)

Question 3

The function f is defined by

$$f(x) = 3 + \sqrt{x - 2} \qquad x \in \mathbb{R} \quad x > 2$$

(a) State the range of f

(1)

(b) Find f^{-1}

(3)

The function g is defined by

$$g(x) = \frac{15}{x - 3} \qquad x \in \mathbb{R} \quad x \neq 3$$

(c) Find gf(6)

(2)

(d) Find the exact value of the constant a for which

$$f(a^2 + 2) = g(a)$$

(2)

Question 4

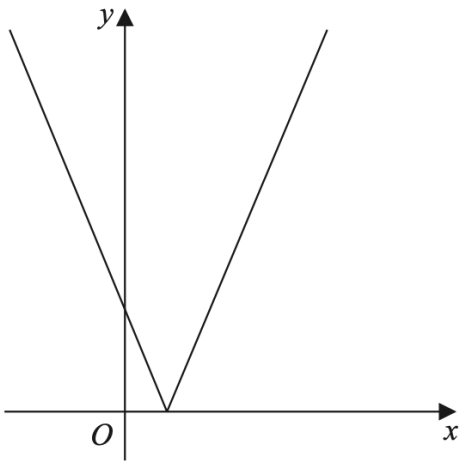


Figure 4

Figure 4 shows a sketch of the graph with equation

$$y = |2x - 3k|$$

where k is a positive constant.

(a) Sketch the graph with equation $y = f(x)$ where

$$f(x) = k - |2x - 3k|$$

stating

- the coordinates of the maximum point
- the coordinates of any points where the graph cuts the coordinate axes

(4)

(b) Find, in terms of k, the set of values of x for which

$$k - |2x - 3k| > x - k$$

giving your answer in set notation.

(4)

(c) Find, in terms of k, the coordinates of the minimum point of the graph with equation

$$y = 3 - 5f\left(\frac{1}{2}x\right)$$

(2)

Question 5

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

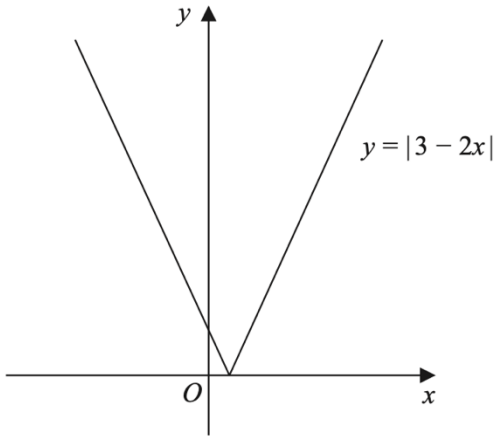


Figure 1

Figure 1 shows a sketch of the graph with equation $y = |3 - 2x|$

Solve

$$|3 - 2x| = 7 + x$$

(4)

Question 5

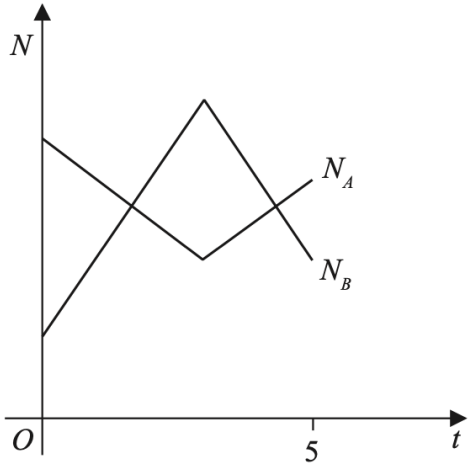


Figure 2

The number of subscribers to two different music streaming companies is being monitored.

The number of subscribers, N_A, in thousands, to **company A** is modelled by the equation

$$N_A = |t - 3| + 4 \qquad t \geqslant 0$$

where t is the time in years since monitoring began.

The number of subscribers, N_B, in thousands, to **company B** is modelled by the equation

$$N_B = 8 - |2t - 6| \qquad t \geqslant 0$$

where t is the time in years since monitoring began.

Figure 2 shows a sketch of the graph of N_A and the graph of N_B over a 5-year period.

Use the equations of the models to answer parts (a), (b), (c) and (d).

(a) Find the initial difference between the number of subscribers to **company A** and the number of subscribers to **company B**.

(2)

When $t = T$ **company A** reduced its subscription prices and the number of subscribers increased.

(b) Suggest a value for T, giving a reason for your answer.

(2)

(c) Find the range of values of t for which $N_A > N_B$ giving your answer in set notation.

(5)

(d) State a limitation of the model used for **company B**.

(1)

Question 6

The function f is defined by

$$f : x \mapsto |2x - 5|, \quad x \in \mathbb{R}$$

(a) Sketch the graph with equation $y = f(x)$, showing the coordinates of the points where the graph cuts or meets the axes.

(2)

(b) Solve $f = (x) = 15 + x$.

(3)

The function g is defined by

$$g : x \mapsto x^2 - 4x + 1, \quad x \in \mathbb{R}, \quad 0 \leq x \leq 5$$

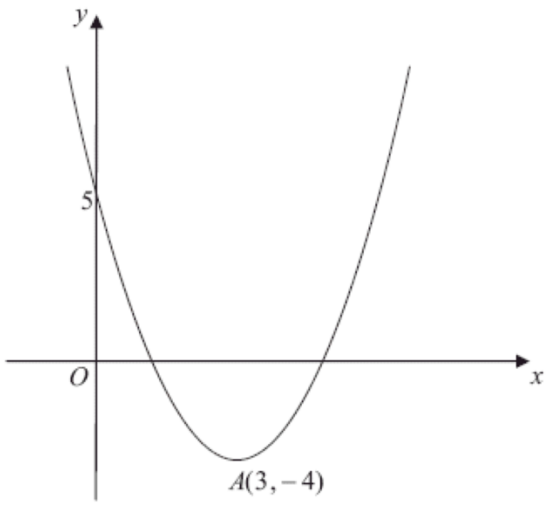
(c) Find fg(2).

(2)

(d) Find the range of g.

(3)
(Total 10 marks)

Question 7



The diagram above shows a sketch of the curve with the equation $y = f(x)$, $x \in \mathbb{R}$.

The curve has a turning point at $A(3, -4)$ and also passes through the point $(0, 5)$.

(a) Write down the coordinates of the point to which A is transformed on the curve with equation

(i) $y = |f(x)|$,

(ii) $y = 2f\left(\frac{1}{2}x\right)$.

(4)

(b) Sketch the curve with equation

$$y = f(|x|)$$

(3)

On your sketch show the coordinates of all turning points and the coordinates of the point at which the curve cuts the y-axis.

The curve with equation $y = f(x)$ is a translation of the curve with equation $y = x^2$.

(c) Find $f(x)$.

(2)

(d) Explain why the function f does not have an inverse.

(1)

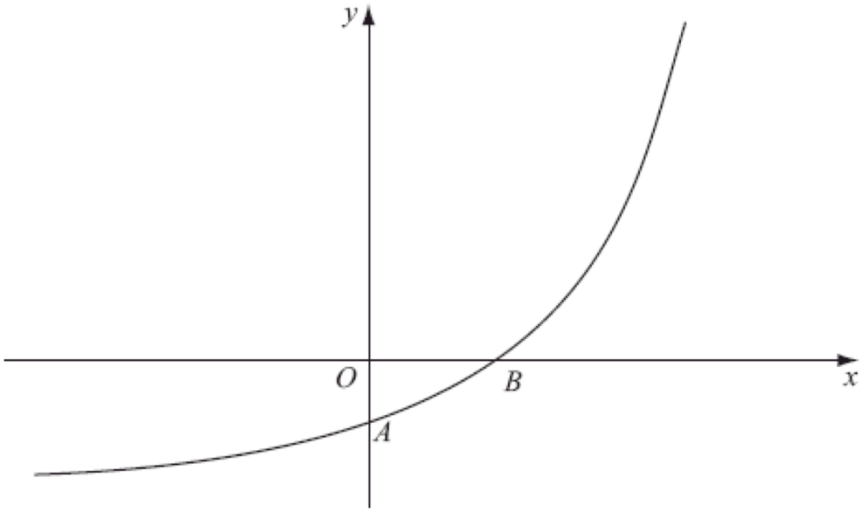
(Total 10 marks)

Question 8

Sketch the graph of $y = \ln|x|$, stating the coordinates of any points of intersection with the axes.

(Total 3 marks)

Question 9



The figure above shows a sketch of part of the curve with equation $y = f(x)$, $x \in \mathbb{R}$

The curve meets the coordinate axes at the points $A(0, 1 - k)$ and $B\left(\frac{1}{2}\ln k, 0\right)$, where k is a constant and $k > 1$, as shown in the diagram above.

On separate diagrams, sketch the curve with equation

(a) $y = |f(x)|$,

(3)

(b) $y = f^{-1}(x)$

(2)

Show on each sketch the coordinates, in terms of k, of each point at which the curve meets or cuts the axes.

Given that $f(x) = e^{2x} - k$,

(c) state the range of f,

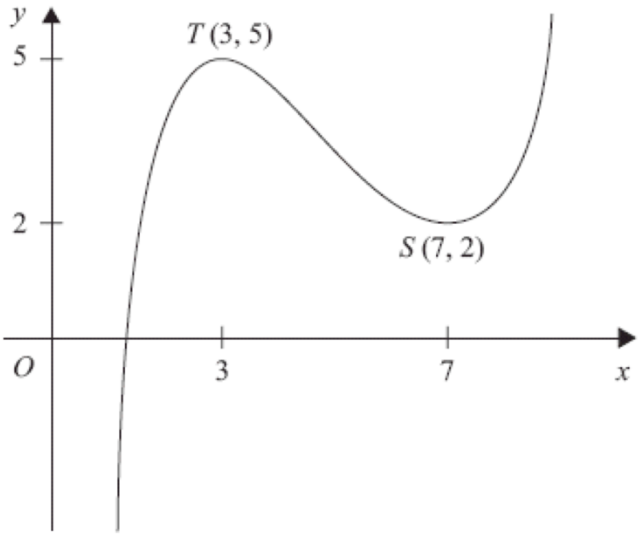
(1)

(d) find $f^{-1}(x)$,

(3)

(e) write down the domain of f^{-1}.

Question 10



The figure above shows the graph of $y = f(x)$, $1 < x < 9$.
The points $T(3, 5)$ and $S(7, 2)$ are turning points on the graph.

Sketch, on separate diagrams, the graphs of

(a) $y = 2f(x) - 4$,

(3)

(b) $y = |f(x)|$.

(3)

Indicate on each diagram the coordinates of any turning points on your sketch.

(Total 6 marks)

Question 11

For the constant k, where $k > 1$, the functions f and g are defined by

$$f: x \mapsto \ln(x + k), \quad x > -k,$$

$$g: x \mapsto |2x - k|, \quad x \in \mathbb{R}.$$

(a) On separate axes, sketch the graph of f and the graph of g.

On each sketch state, in terms of k, the coordinates of points where the graph meets the coordinate axes.

(5)

(b) Write down the range of f.

(1)

(c) Find $\text{fg}\left(\frac{k}{4}\right)$ in terms of k, giving your answer in its simplest form.

(2)

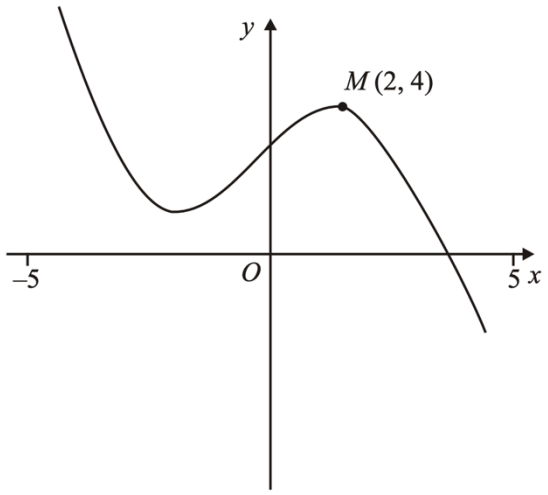
The curve C has equation $y = \text{f}(x)$. The tangent to C at the point with x-coordinate 3 is parallel to the line with equation $9y = 2x + 1$.

(d) Find the value of k.

(4)

(Total 12 marks)

Question 12



The figure above shows the graph of $y = f(x)$, $-5 \leq x \leq 5$.
The point M (2, 4) is the maximum turning point of the graph.

Sketch, on separate diagrams, the graphs of

(a) $y = \text{f}(x) + 3$,

(2)

(b) $y = |\text{f}(x)|$,

(2)

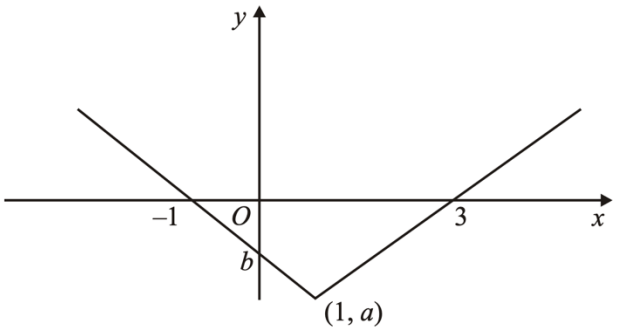
(c) $y = \text{f}(|x|)$.

(3)

Show on each graph the coordinates of any maximum turning points.

(Total 7 marks)

Question 13



This figure shows part of the graph of $y = f(x)$, $x \in \mathbb{R}$. The graph consists of two line segments that meet at the point $(1, a)$, $a < 0$. One line meets the x-axis at $(3, 0)$. The other line meets the x-axis at $(–1, 0)$ and the y-axis at $(0, b)$, $b < 0$.

In separate diagrams, sketch the graph with equation

(a) $y = f(x + 1)$,

(2)

(b) $y = f(|x|)$.

(3)

Indicate clearly on each sketch the coordinates of any points of intersection with the axes.

Given that $f(x) = |x – 1| – 2$, find

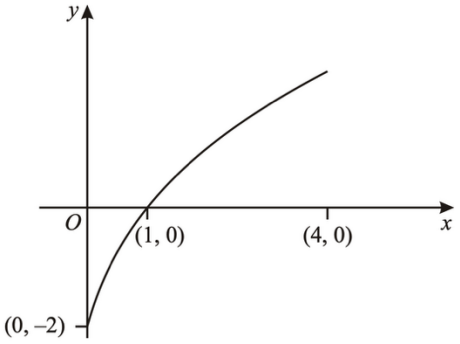
(c) the value of a and the value of b,

(2)

(d) the value of x for which $f(x) = 5x$.

(4)
(Total 11 marks)

Question 14



This diagram shows a sketch of the curve with equation $y = f(x)$, $0 \leq x \leq 4$. The curve passes through the point $(1, 0)$ on the x-axis and meets the y-axis at the point $(0, –2)$.

Sketch, on separate axes, the graph of

(a) $y = |f(x)|$,

(2)

(b) $y = f(2x)$,

(2)

(c) $y = f^{-1}(x)$,

(3)

in each case showing the coordinates of the points at which the graph meets the axes.

(Total 7 marks)

Question 15

The function f is defined by

$$f: x \mapsto |2x - 5|, \quad x \in \mathbb{R}.$$

(a) Sketch the graph of $y = f(x)$, showing the coordinates of points at which the graph meets or crosses the axes.

(3)

(b) Find the values of x for which $f(x) = x$.

(4)

The function g is defined by

$$g: x \mapsto x(x - 6), \quad x \in \mathbb{R}.$$

(c) Find the range of $g(x)$.

(3)

(d) Find fg(1).

(2)

(Total 12 marks)

Question 16

The functions f and g are defined by

$$f: x \mapsto |x - a| + a, \quad x \in \mathrm{R},$$

$$g: x \mapsto 4x + a, \quad x \in \mathrm{R}.$$

where a is a positive constant.

(a) On the same diagram, sketch the graphs of f and g, showing clearly the coordinates of any points at which your graphs meet the axes.

(5)

(b) Use algebra to find, in terms of a, the coordinates of the point at which the graphs of f and g intersect.

(3)

(c) Find an expression for $\text{fg}(x)$.

(2)

(d) Solve, for x in terms of a, the equation

$$\text{fg}(x) = 3a.$$

(3)
(Total 13 marks)

Question 17

(a) Sketch the graph of $y = |2x + a|$, $a > 0$, showing the coordinates of the points where the graph meets the coordinate axes.

(2)

(b) On the same axes, sketch the graph of $y = \dfrac{1}{x}$.

(1)

(c) Explain how your graphs show that there is only one solution of the equation

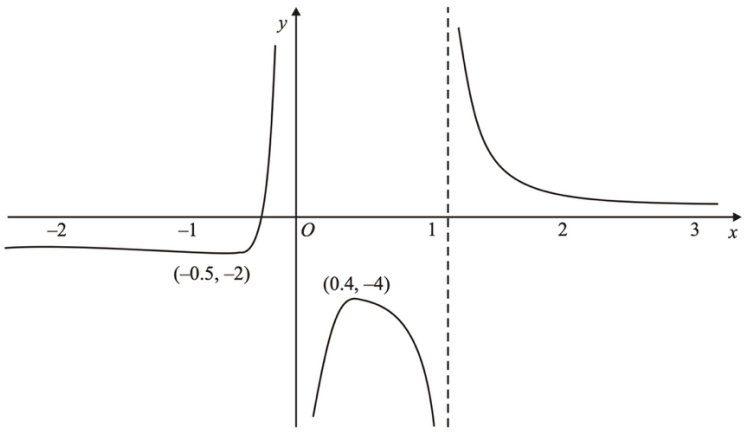
$$x|2x + a| - 1 = 0.$$

(1)

(d) Find, using algebra, the value of x for which $x|2x + 1| - 1 = 0$.

(3)
(Total 7 marks)

Question 18



On a separate diagram sketch the graphs of

(a) $y = |f(x)|$,

(4)

(b) $y = f(x - 3)$,

(4)

(c) $y = f(|x|)$.

(4)

In each case show clearly

(i) the coordinates of any points at which the curve has a maximum or minimum point,

(ii) how the curve approaches the asymptotes of the curve.

(Total 12 marks)

The diagram above shows a sketch of part of the curve with equation $y = f(x)$, $x \in \mathbb{R}$.

The curve has a minimum point at $(-0.5, -2)$ and a maximum point at $(0.4, -4)$. The lines $x = 1$, the x-axis and the y-axis are asymptotes of the curve, as shown in the diagram above.