

| Q | Scheme  | Marks | AOs  | Pearson Progression Step and Progress descriptor |
|---|---|-------|------|--|
| 1 | Makes an attempt to factor all the quadratics on the left-hand side of the identity.  | M1    | 2.2a | 5th Simplify algebraic fractions.                |
|   | Correctly factors each expression on the left-hand side of the identity:  | A1    | 2.2a |  |
|   | $\frac{(x-6)(x+6)}{(x-5)(x-6)} \times \frac{(5-x)(5+x)}{Ax^2 + Bx + C} \times \frac{(3x-1)(2x+3)}{(3x-1)(x+6)}$   |       |      |  |
|   | Successfully cancels common factors:<br>$\frac{(-1)(5+x)(2x+3)}{Ax^2 + Bx + C} \equiv \frac{x+5}{(-1)(x-6)}$  | M1    | 1.1b |  |
|   | States that $Ax^2 + Bx + C \equiv (2x+3)(x-6)$  | M1    | 1.1b |  |
|   | States or implies that $A = 2$ , $B = -9$ and $C = -18$   | A1    | 1.1b |  |
|   | <b>(5 marks)</b>  |       |      |  |
|   | <p><b>Notes</b></p> <p><b>Alternative method</b></p> <p>Makes an attempt to substitute <math>x = 0</math> (M1)</p> <p>Finds <math>C = -18</math> (A1)</p> <p>Substitutes <math>x = 1</math> to give <math>A + B = -7</math> (M1)</p> <p>Substitutes <math>x = -1</math> to give <math>A - B = 11</math> (M1)</p> <p>Solves to get <math>A = 2</math>, <math>B = -9</math> and <math>C = -18</math> (A1)</p> |       |      |  |

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|---|---|-------|------|--|--|
| 2 | <p>Correctly factorises the denominator of the left-hand fraction:</p> $\frac{6}{(2x+5)(2x-1)} + \frac{3x+1}{2x-1}$   | M1    | 2.2a | 4th<br>Add, subtract, multiply and divide algebraic fractions. |  |
|   | <p>Multiplies the right-hand fraction by <math>\frac{2x+5}{2x+5}</math></p> <p>For example:</p> $\frac{6}{(2x+5)(2x-1)} + \frac{(3x+1)(2x+5)}{(2x-1)(2x+5)} \text{ is seen.}$ | M1    | 1.1b |  |  |
|   | <p>Makes an attempt to distribute the numerator of the right-hand fraction.</p> <p>For example:</p> $\frac{6 + 6x^2 + 17x + 5}{(2x+5)(2x-1)} \text{ is seen.}$                | M1    | 1.1b |  |  |
|   | <p>Fully simplified answer is seen.</p> <p>Accept either <math>\frac{6x^2 + 17x + 11}{(2x+5)(2x-1)}</math> or <math>\frac{(6x+11)(x+1)}{(2x+5)(2x-1)}</math></p>              | A1    | 1.1b |  |  |
|   |   |       |      | (4 marks)  |  |
|   | <b>Notes</b>  |       |      |  |  |

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|--|--|------------|------|---|--|
| <b>3</b>   | States that:<br>$A(2x+5) + B(5x-1) \equiv 6x+42$   | <b>M1</b>  | 2.2a | 5th<br>Decompose algebraic fractions into partial fractions – two linear factors. |  |
|  | Equates the various terms.<br>Equating the coefficients of $x$ : $2A+5B=6$<br>Equating constant terms: $5A-B=42$ | <b>M1*</b> | 2.2a |   |  |
|  | Multiplies both of the equations in an effort to equate one of the two variables.                                | <b>M1*</b> | 1.1b |   |  |
|  | Finds $A = 8$  | <b>A1</b>  | 1.1b |   |  |
|  | Find $B = -2$  | <b>A1</b>  | 1.1b |   |  |
| <b>(5 marks)</b>   |  |            |      |   |  |
| <p style="text-align: center;"><b>Notes</b></p> <p><b>Alternative method</b></p> <p>Uses the substitution method, having first obtained this equation: <math>A(2x+5) + B(5x-1) \equiv 6x+42</math></p> <p>Substitutes <math>x = -\frac{5}{2}</math> to obtain <math>-\frac{27}{2}B = 27</math> (<b>M1</b>)</p> <p>Substitutes <math>x = \frac{1}{5}</math> to obtain <math>\frac{27}{5}A = 43.2</math> (<b>M1</b>)</p> |  |            |      |   |  |

| Q         | Scheme   | Marks | AOs  | Pearson Progression Step and Progress descriptor                                    |
|-----------|--|-------|------|---|
| 4         | States that:<br>$A(4-x)(x+5) + B(x-3)(x+5) + C(x-3)(4-x) \equiv 4x^2 + x - 23$   | M1    | 2.2a | 6th<br>Decompose algebraic fractions into partial fractions – three linear factors. |
|           | Further states that:<br>$A(-x^2 - x + 20) + B(x^2 + 2x - 15) + C(-x^2 + 7x - 12) \equiv 4x^2 + x - 23$   | M1    | 1.1b |   |
|           | Equates the various terms.<br>Equating the coefficients of $x^2$ : $-A + B - C = 4$<br>Equating the coefficients of $x$ : $-A + 2B + 7C = 1$<br>Equating constant terms: $20A - 15B - 12C = -23$ | M1*   | 2.2a |   |
|           | Makes an attempt to manipulate the expressions in order to find $A$ , $B$ and $C$ . Obtaining two different equations in the same two variables would constitute an attempt.                     | M1*   | 1.1b |   |
|           | Finds the correct value of any one variable:<br>either $A = 2$ , $B = 5$ or $C = -1$   | A1*   | 1.1b |   |
|           | Finds the correct value of all three variables:<br>$A = 2$ , $B = 5$ , $C = -1$  | A1    | 1.1b |   |
| (6 marks) |  |       |      |   |

### Notes

#### Alternative method

Uses the substitution method, having first obtained this equation:

$$A(4-x)(x+5) + B(x-3)(x+5) + C(x-3)(4-x) \equiv 4x^2 + x - 23$$

Substitutes  $x = 4$  to obtain  $9B = 45$  (M1)

Substitutes  $x = 3$  to obtain  $8A = 16$  (M1)

Substitutes  $x = -5$  to obtain  $-72C = 72$  (A1)

| Q | Scheme   | Marks | AOs  | Pearson Progression Step and Progress descriptor                                |  |
|---|--|-------|------|---|--|
| 5 | States that:<br>$A(x-4)(3x+1) + B(3x+1) + C(x-4)(x-4) \equiv 18x^2 - 98x + 78$   | M1    | 2.2a | 7th<br>Decompose algebraic fractions into partial fractions – repeated factors. |  |
|   | Further states that:<br>$A(3x^2 - 11x - 4) + B(3x+1) + C(x^2 - 8x + 16) \equiv 18x^2 - 98x + 78$   | M1    | 1.1b |   |  |
|   | Equates the various terms.<br>Equating the coefficients of $x^2$ : $3A + C = 18$<br>Equating the coefficients of $x$ : $-11A + 3B - 8C = -98$<br>Equating constant terms: $-4A + B + 16C = 78$   | M1    | 2.2a |   |  |
|   | Makes an attempt to manipulate the expressions in order to find $A$ , $B$ and $C$ . Obtaining two different equations in the same two variables would constitute an attempt.   | M1    | 1.1b |   |  |
|   | Finds the correct value of any one variable:<br>either $A = 4$ , $B = -2$ or $C = 6$   | A1    | 1.1b |   |  |
|   | Finds the correct value of all three variables:<br>$A = 4$ , $B = -2$ , $C = 6$  | A1    | 1.1b |   |  |
|   |  |       |      | (6 marks)   |  |
|   | <b>Notes</b><br><b>Alternative method</b><br>Uses the substitution method, having first obtained this equation:<br>$A(x-4)(3x+1) + B(3x+1) + C(x-4)(x-4) \equiv 18x^2 - 98x + 78$ Substitutes $x = 4$ to obtain $13B = -26$<br>Substitutes $x = -\frac{1}{3}$ to obtain $\frac{169}{9}C = \frac{338}{3} \Rightarrow C = \frac{1014}{169} = 6$<br>Equates the coefficients of $x^2$ : $3A + C = 18$<br>Substitutes the found value of $C$ to obtain $3A = 12$ |       |      |   |  |

| Q | Scheme   | Marks | AOs  | Pearson Progression Step and Progress descriptor                  |
|---|--|-------|------|---|
| 6 | <p>Makes an attempt to set up a long division.</p> <p>For example: <math>x + 6 \overline{)x^3 + 8x^2 - 9x + 12}</math> is seen.</p> <p>Award 1 accuracy mark for each of the following:<br/> <math>x^2</math> seen, <math>2x</math> seen, <math>-21</math> seen.</p> <p>For the final accuracy mark either <math>D = 138</math> or <math>\frac{138}{x + 6}</math> or the remainder is 138 must be seen.</p> $  \begin{array}{r}  x^2 + 2x - 21 \\  x + 6 \overline{)x^3 + 8x^2 - 9x + 12} \\  \underline{x^3 + 6x^2} \\  \phantom{x^3 + 6x^2} 2x^2 - 9x \\  \phantom{x^3 + 6x^2} \underline{2x^2 + 12x} \\  \phantom{x^3 + 6x^2 + 2x^2} -21x + 12 \\  \phantom{x^3 + 6x^2 + 2x^2} \underline{-21x - 126} \\  \phantom{x^3 + 6x^2 + 2x^2 - 21x} 138  \end{array}  $ | M1    | 2.2a | 5th<br>Divide polynomials by linear expressions with a remainder. |

(5 marks)

**Notes**

This question can be solved by first writing  $(Ax^2 + Bx + C)(x + 6) + D \equiv x^3 + 8x^2 - 9x + 12$  and then solving for  $A, B, C$  and  $D$ . Award 1 mark for the setting up the problem as described. Then award 1 mark for each correct coefficient found. For example:

Equating the coefficients of  $x^3$ :  $A = 1$

Equating the coefficients of  $x^2$ :  $6 + B = 8$ , so  $B = 2$

Equating the coefficients of  $x$ :  $12 + C = -9$ , so  $C = -21$

Equating the constant terms:  $-126 + D = 12$ , so  $D = 138$ .

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|---|--|-------|------|---|--|
| 7 | <p>Makes an attempt to set up a long division.</p> <p>For example: <math>x^2 - 2x - 15 \overline{)x^4 + 2x^3 - 29x^2 - 48x + 90}</math> is seen.</p>   | M1    | 2.2a | 6th<br>Decompose algebraic fractions into partial fractions – three linear factors. |  |
|   | <p>Award 1 accuracy mark for each of the following:</p> <p><math>x^2</math> seen, <math>4x</math> seen, <math>-6</math> seen.</p> $  \begin{array}{r}  x^2 + 4x - 6 \\  x^2 - 2x - 15 \overline{)x^4 + 2x^3 - 29x^2 - 47x + 77} \\  \underline{x^4 - 2x^3 - 15x^2} \\  4x^3 - 14x^2 - 47x \\  \underline{4x^3 - 8x^2 - 60x} \\  -6x^2 + 13x + 77 \\  \underline{-6x^2 + 12x + 90} \\  x - 13  \end{array}  $ | A3    | 1.1b |   |  |
|   | <p>Equates the various terms to obtain the equation:</p> <p><math>x - 13 = V(x - 5) + W(x + 3)</math></p> <p>Equating the coefficients of <math>x</math>: <math>V + W = 1</math></p> <p>Equating constant terms: <math>-5V + 3W = -13</math></p>   | M1    | 2.2a |   |  |
|   | Multiplies one or or both of the equations in an effort to equate one of the two variables.  | M1    | 1.1b |   |  |
|   | Finds $W = -1$ and $V = 2$ .   | A1    | 1.1b |   |  |
|   |  |       |      | (7 marks)   |  |
|   | Notes  |       |      |   |  |

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|------------------|--|-----------|------|---|--|
| <b>8</b>         | Equating the coefficients of $x^4$ : $A = 5$                 | <b>A1</b> | 2.2a | 6th<br>Solve problems using the remainder theorem linked to improper algebraic fractions. |  |
|                  | Equating the coefficients of $x^3$ : $B = -4$                | <b>A1</b> | 1.1b |   |  |
|                  | Equating the coefficients of $x^2$ : $2A + C = 17$ , $C = 7$ | <b>A1</b> | 1.1b |   |  |
|                  | Equating the coefficients of $x$ : $2B + D = -5$ , $D = 3$   | <b>A1</b> | 1.1b |   |  |
|                  | Equating constant terms: $2C + E = 7$ , $E = -7$             | <b>A1</b> | 1.1b |   |  |
| <b>(5 marks)</b> |  |           |      |   |  |
| <b>Notes</b>     |  |           |      |   |  |

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|---|---|-------|------|--|--|
| 9 | <p>Makes an attempt to set up a long division.</p> <p>For example: <math>9x^2 + 0x - 16 \overline{)9x^2 + 25x + 16}</math> is seen.</p> <p>The '0x' being seen is not necessary to award the mark.</p>  | M1    | 2.2a | <p>5th</p> <p>Decompose algebraic fractions into partial fractions – two linear factors.</p> |  |
|   | <p>Long division completed so that a '1' is seen in the quotient and a remainder of <math>25x + 32</math> is also seen.</p> $  \begin{array}{r}  1 \\  9x^2 + 0x - 16 \overline{)9x^2 + 25x + 16} \\  9x^2 + 0x - 16 \\  \hline  25x + 32  \end{array}  $ | M1    | 1.1b |  |  |
|   | <p>States <math>B(3x + 4) + C(3x - 4) \equiv 25x - 32</math></p>  | M1    | 1.1b |  |  |
|   | <p>Equates the various terms.</p> <p>Equating the coefficients of <math>x</math>: <math>3B + 3C = 25</math></p> <p>Equating constant terms: <math>4B - 4C = 32</math></p>   | M1    | 2.2a |  |  |
|   | <p>Multiplies one or both of the equations in an effort to equate one of the two variables.</p>   | M1    | 1.1b |  |  |
|   | <p>Finds <math>B = \frac{49}{6}</math></p>  | A1    | 1.1b |  |  |
|   | <p>Finds <math>C = \frac{1}{6}</math></p>   | A1    | 1.1b |  |  |

(7 marks)

## Notes

## Alternative method

Writes  $A + \frac{B}{3x-4} + \frac{C}{3x+4}$  as  $\frac{A(3x-4)(3x+4)}{9x^2-16} + \frac{B(3x+4)}{9x^2-16} + \frac{C(3x-4)}{9x^2-16}$

States  $A(3x-4)(3x+4) + B(3x+4) + C(3x-4) \equiv 9x^2 + 25x + 16$

Substitutes  $x = \frac{4}{3}$  to obtain:  $8B = \frac{196}{3} \Rightarrow B = \frac{49}{6}$

Substitutes  $x = -\frac{4}{3}$  to obtain:  $-8C = -\frac{4}{3} \Rightarrow C = \frac{1}{6}$

Equating the coefficients of  $x^2$ :  $9A = 9 \Rightarrow A = 1$