A level Mathematics Practice Paper – Algebra and functions – Mark scheme

Question	Scheme	Marks
1	$$\begin{array}{r} 3x^2 - 2x + 7 \\ x^2(+0x) - 4 \overline{)\,3x^4 - 2x^3 - 5x^2 + (0x) - 4} \\ \underline{3x^4 + 0x^3 - 12x^2} \\ -2x^3 + 7x^2 + 0x \\ \underline{-2x^3 + 0x^2 + 8x} \\ 7x^2 - 8x - 4 \\ \underline{7x^2 + 0x - 28} \\ -8x + 24 \end{array}$$	
	$a = 3$	B1
	Long division as far as $$\begin{array}{r} 3x^2 - 2x..... \\ x^2(+0x) - 4 \overline{)\,3x^4 - 2x^3 - 5x^2 + (0x) - 4} \\ \underline{3x^4 + 0x^3 - 12x^2} \\ -2x^3 + ... \\ \underline{-2x^3 + ...} \end{array}$$	M1
	Two of $b = -2 \quad c = 7 \quad d = -8 \quad e = 24$	A1
	All four of $b = -2 \quad c = 7 \quad d = -8 \quad e = 24$	A1
		(4 marks)
2	$x^2 - 9 = (x+3)(x-3)$	B1
	$\dfrac{4x}{x^2-9} - \dfrac{2}{(x+3)} = \dfrac{4x-2(x-3)}{(x+3)(x-3)}$	M1
	$= \dfrac{2x+6}{(x+3)(x-3)}$	A1
	$= \dfrac{2\cancel{(x+3)}}{\cancel{(x+3)}(x-3)}$	
	$= \dfrac{2}{(x-3)}$	A1
		(4 marks)

PJ@SELEXIQ

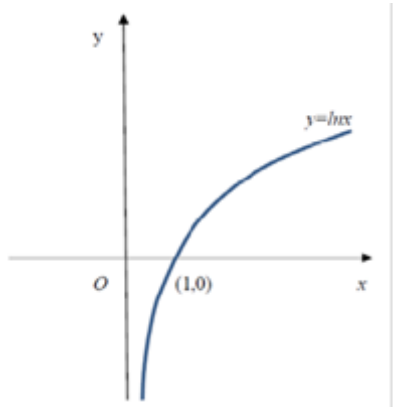
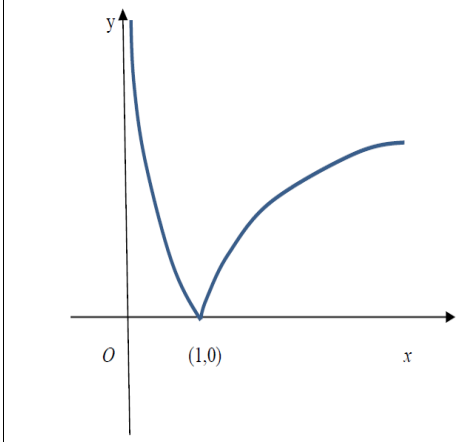
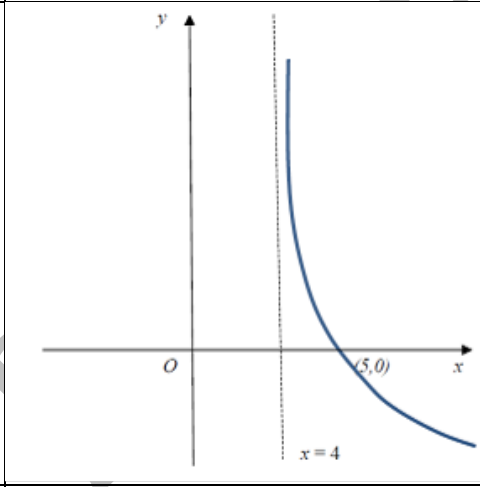
Question	Scheme	Marks
3	Factorise $4x^2-9=(2x-3)(2x+3)$	B1
	Use of common denominator $\frac{3}{2x+3}-\frac{1}{2x-3}+\frac{6}{4x^2-9}=\frac{3(2x-3)-1(2x+3)+6}{(2x+3)(2x-3)}$	M1
	$=\frac{4x-6}{(2x+3)(2x-3)}$	A1
	$=\frac{2\cancel{(2x-3)}}{(2x+3)\cancel{(2x-3)}}=\frac{2}{2x+3}$	A1
		(4 marks)
4	$9x^2-4=(3x-2)(3x+2)$ At any stage	B1
	Eliminating the common factor of $(3x+2)$ at any stage	
	$\frac{2\cancel{(3x+2)}}{(3x-2)\cancel{(3x+2)}}=\frac{2}{3x-2}$	B1
	Use of a common denominator	
	$\frac{2(3x+2)(3x+1)}{(9x^2-4)(3x+1)}-\frac{2(9x^2-4)}{(9x^2-4)(3x+1)}$ or $\frac{2(3x+1)}{(3x-2)(3x+1)}-\frac{2(3x-2)}{(3x+1)(3x-2)}$	M1
	$\frac{6}{(3x-2)(3x+1)}$ or $\frac{6}{9x^2-3x-2}$	A1
		(4 marks)
5(a)	$\text{fg}(x)=\frac{28}{x-2}-1$	M1
	$\left(=\frac{30-x}{x-2}\right)$	
	Sets $\text{fg}(x)=x\Rightarrow\frac{28}{x-2}-1=x$	
	$\Rightarrow 28=(x+1)(x-2)$	M1
	$\Rightarrow x^2-x-30=0$	
	$\Rightarrow (x-6)(x+5)=0$	dM1
	$\Rightarrow x=6\,,\,x=-5$	A1
		(4)
5(b)	$a=6$	B1ft
		(1)

PJ@SELEXIQ

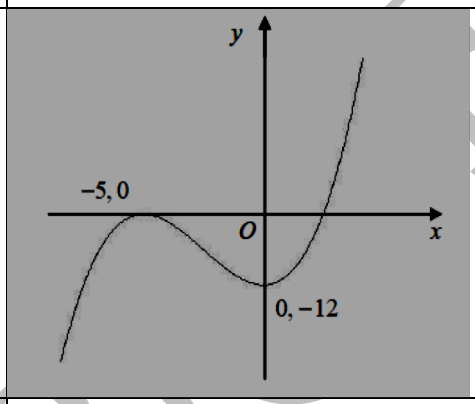
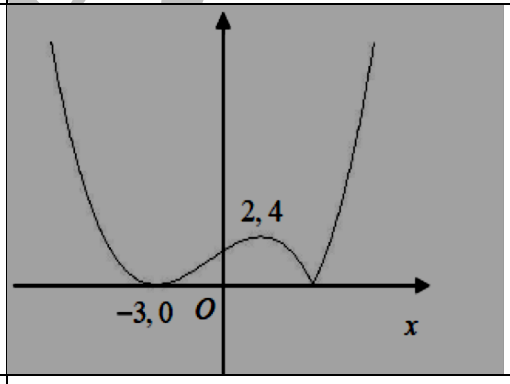
A level Mathematics Practice Paper – Algebra and functions – Mark scheme

Question	Scheme	Marks
		(5 marks)

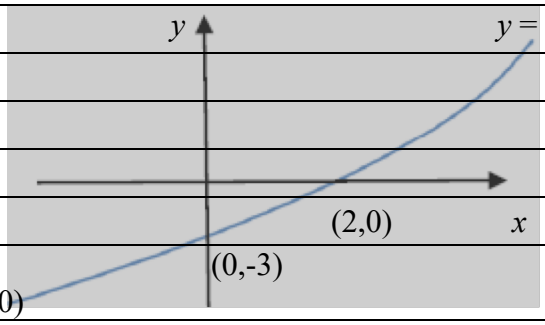
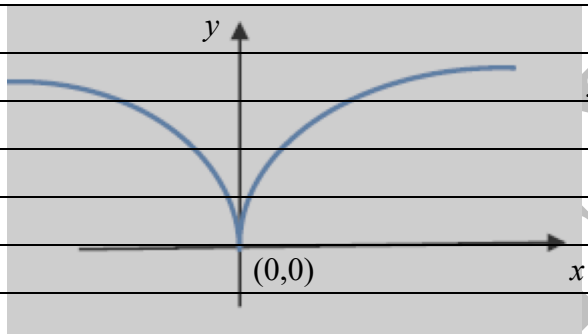
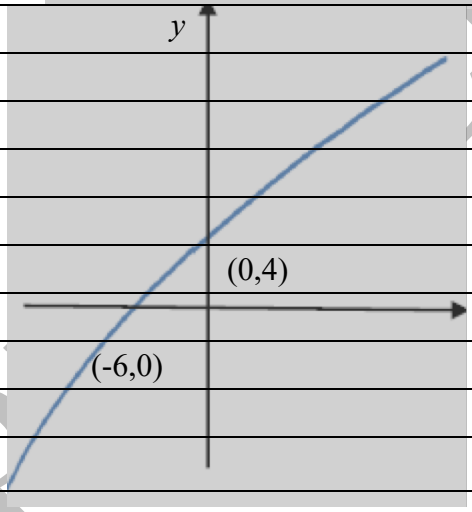
PJ@SELEXIQ

Question	Scheme	Marks
6	 *ln* graph crossing x axis at (1,0) and asymptote at $x = 0$	B1
	 Shape including cusp Touches or crosses the x axis at (1,0) Asymptote given as x=0	B1ft B1ft B1
	 Shape Crosses at (5, 0) Asymptote given as $x = 4$	B1 B1ft B1
		(7 marks)

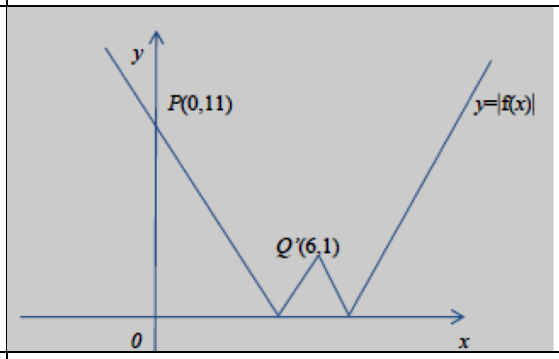
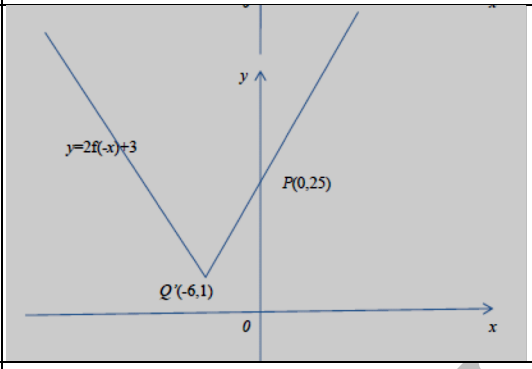
PMTEXIQ

Question	Scheme	Marks
7(a)	$y \geq 3$	B1
		(1)
7(b)	$y = 3 + \sqrt{x+2} \Rightarrow y - 3 = \sqrt{x+2} \Rightarrow x = (y-3)^2 - 2$	M1 A1
	$\Rightarrow g^{-1}(x) = (x-3)^2 - 2, \quad \text{with } x \geq 3$	A1
		(3)
7(c)	$g(x) = x \Rightarrow 3 + \sqrt{x+2} = x$	
	$\Rightarrow x + 2 = (x-3)^2 \Rightarrow x^2 - 7x + 7 = 0$	M1 A1
	$\Rightarrow x = \frac{7 \pm \sqrt{21}}{2} \Rightarrow x = \frac{7+\sqrt{21}}{2}$ only	M1 A1
		(4)
7(d)	$a = \frac{7+\sqrt{21}}{2}$	B1 ft
		(1)
		(9 marks)
8(a)	 Shape x coordinates correct y coordinates correct	B1 B1 B1
		(3)
8(b)	 Shape Max at (2,4) Min at (-3,0)	B1 B1 B1
		(3)
		(6 marks)

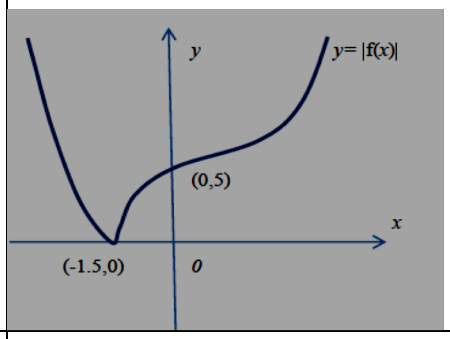
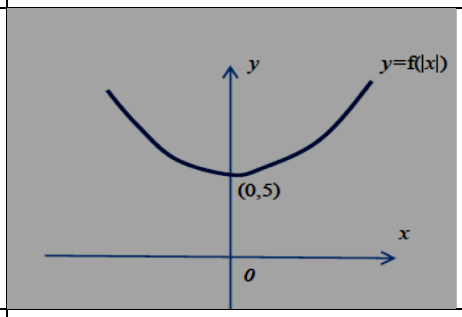
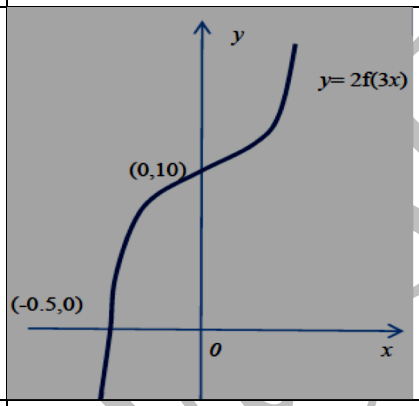
PHYSICSLEXIQ

Question	Scheme	Marks
9(a)	ff(-3)= f(0),=2	M1 A1
		(2)
9(b)		
	Shape	B1
	(0,-3) and	B1
		(2)
9(c)		
	Shape	B1
	(0,0)	B1
		(2)
9(d)		
	Shape	B1
	(-6,0) or (0,4)	B1
	(-6,0) and (0,4)	B1
		(3)
		(9 marks)

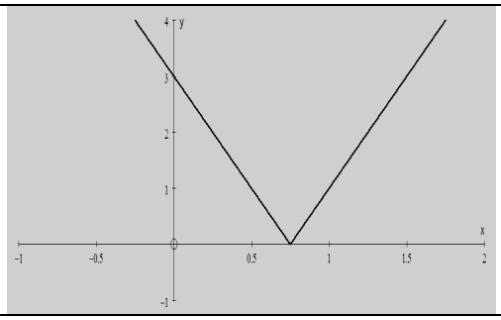
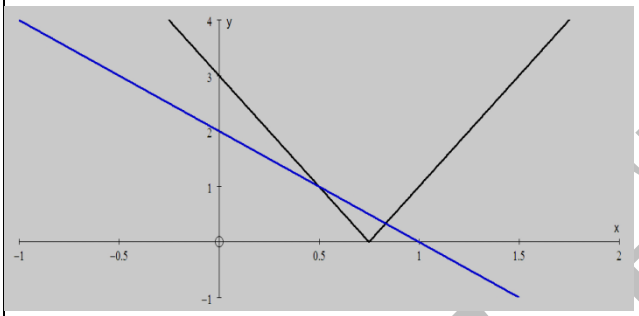
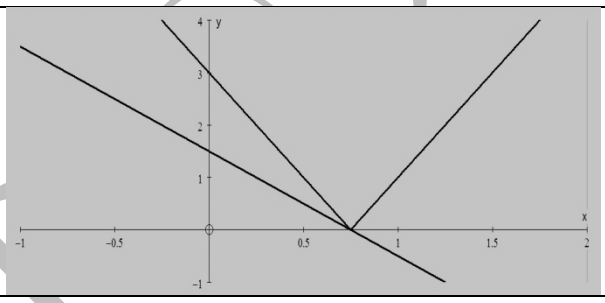
PEARSON EDEXCEL

Question	Scheme	Marks
10(a)	 'W' Shape (0, 11) and (6, 1)	B1 B1
		(2)
10(b)	 'V' shape (-6,1) (0,25)	B1 B1 B1
		(3)
10(c)	One of $a = 2$ or $b = 6$	B1
	$a = 2$ and $b = 6$	B1
		(2)
		(7 marks)

PJ@SELEXIQ

Question	Scheme	Marks
11(a)	 Shape including cusp (–1.5, 0) **and** (0, 5)	B1 B1
		(2)
11(b)	 Shape (0,5)	B1 B1
		(2)
11(c)	 Shape (0,10) (-0.5, 0)	B1 B1 B1
		(3)
		(7 marks)

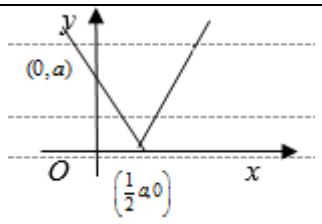
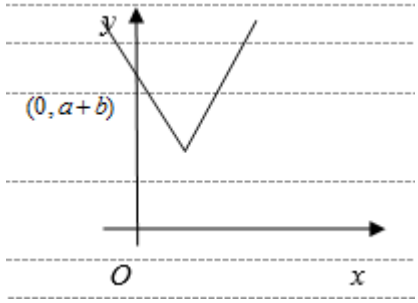
PJC SELEXIQ

Question	Scheme	Marks
12(a)	 V shaped graph Touches x axis at ¾ and cuts y axis at 3	B1 B1
		(2)
12(b)	 Solves $4x-3=2-2x$ or $3-4x=2-2x$ to give either value of x Both $x=\frac{5}{6}$ and $x=\frac{1}{2}$ or $x>\frac{5}{6}$ or $x<\frac{1}{2}$	M1 A1
	$x<\frac{1}{2}$ or $x>\frac{5}{6}$	dM1A1
		(4)
12(c)	 Draws graph Or solves $\lvert 4x-3\rvert=1\frac{1}{2}-2x$ to give one soln x = ¾	M1
	Accept for all values of x except $x=\frac{3}{4}$ Or $(x\in\tilde{}\,,)\ x\neq\frac{3}{4}$, or $x<\frac{3}{4}$, $x>\frac{3}{4}$	A1
		(2)
		(8 marks)

PEARSON EDEXCEL

Question	Scheme	Marks
13(a)	$x^2+x-6=(x+3)(x-2)$	B1
	$\frac{x}{x+3}+\frac{3(2x+1)}{(x+3)(x-2)}=\frac{x(x-2)+3(2x+1)}{(x+3)(x-2)}$	M1
	$=\frac{x^2+4x+3}{(x+3)(x-2)}$	A1
	$=\frac{(x+3)(x+1)}{(x+3)(x-2)}$	
	$=\frac{(x+1)}{(x-2)}$ cso	A1*
		(4)
13(b)	One end either $(y)>1,(y)\ddot{O}1$ or $(y)<4,(y)\tilde{N}4$	B1
	$1<y<4$	B1
		(2)
13(c)	Attempt to set	
	Either $g(x)=x$ or $g(x)=g^{-1}(x)$ or $g^{-1}(x)=x$ or $g^2(x)=x$	
	$\frac{(x+1)}{(x-2)}=x$ $\frac{x+1}{x-2}=\frac{2x+1}{x-1}$ $\frac{2x+1}{x-1}=x$ $\frac{\frac{x+1}{x-2}+1}{\frac{x+1}{x-2}-2}=x$	M1
	$x^2-3x-1=0\Rightarrow x=...$	A1, dM1
	$a=\frac{3+\sqrt{13}}{2}$ oe $\left(1.5+\sqrt{3.25}\right)$ cso	A1
		(4)
		(10 marks)

PJ@SELEXIO

Question	Scheme	Marks
14(a)(i)	 V shape on x - axis **or** coordinates $\left(\frac{1}{2}a,0\right)$ **and** $(0,a)$ Correct shape, position and coordinates	B1 B1
14(a)(ii)	 Their "V" shape translated up or $(0,a+b)$ Correct shape, position and $(0,a+b)$	B1ft B1
		(4)
14(b)	States or uses $a+b=8$	B1
	Attempts to solve $\lvert 2x-a\rvert+b=\frac{3}{2}x+8$ in either x or with $x=c$	
	$2c-a+b=\frac{3}{2}c+8\Rightarrow kc=\text{f}(a,b)$	M1
	Combines $kc=\text{f}(a,b)$ with $a+b=8 \quad \Rightarrow c=4a$	dM1 A1
		(4)
		(8 marks)
15	$9x^2=A(x-1)(2x+1)+B(2x+1)+C(x-1)^2$	B1
	$x\to 1 \qquad 9=3B \Rightarrow B=3$	M1
	$x\to -\frac{1}{2} \qquad \frac{9}{4}=\left(-\frac{3}{2}\right)^2 C \Rightarrow C=1$ Any two of A, B, C	A1
	x^2 terms $\qquad 9=2A+C \Rightarrow A=4$ All three correct	A1
		(4 marks)

PJ@SELEXIQ

Question	Scheme	Marks
16	**Method 1: Using one identity**	
	$\frac{9x^2+20x-10}{(x+2)(3x-1)} \equiv A + \frac{B}{(x+2)} + \frac{C}{(3x-1)}$	
	$A = 3$	
	their constant term $= 3$	B1
	$9x^2+20x-10 \equiv A(x+2)(3x-1)+B(3x-1)+C(x+2)$	B1
	Forming a correct identity.	
	Either x^2: $9 = 3A$, x: $20 = 5A + 3B + C$ constant: $-10 = -2A - B + 2C$ or $x = -2 \Rightarrow 36 - 40 - 10 = -7B \Rightarrow -14 = -7B \Rightarrow B = 2$ $x = \frac{1}{3} \Rightarrow 1 + \frac{20}{3} - 10 = \frac{7}{3}C \Rightarrow -\frac{7}{3} = \frac{7}{3}C \Rightarrow C = -1$	
	Attempts to find the value of either one of their B or their C from their identity.	M1
	Correct values for their B and their C, which are found using a correct identity.	A1
		(4)
	Method 2: Long Division	
	$\frac{9x^2+20x-10}{(x+2)(3x-1)} \equiv 3 + \frac{5x-4}{(x+2)(3x-1)}$	
	their constant term $= 3$	B1
	So, $\frac{5x-4}{(x+2)(3x-1)} \equiv \frac{B}{(x+2)} + \frac{C}{(3x-1)}$	
	$5x - 4 \equiv B(3x-1) + C(x+2)$	
	Forming a correct identity.	B1
	Either x: $5 = 3B + C$, constant: $-4 = -B + 2C$ or $x = -2 \Rightarrow -10 - 4 = -7B \Rightarrow -14 = -7B \Rightarrow B = 2$ $x = \frac{1}{3} \Rightarrow \frac{5}{3} - 4 = \frac{7}{3}C \Rightarrow -\frac{7}{3} = \frac{7}{3}C \Rightarrow C = -1$	
	Attempts to find the value of either one of their B or their C from their identity.	M1

PI@SELEXIQ

Question	Scheme	Marks
	Correct values for their B and their C, which are found using $5x-4 \equiv B(3x-1)+C(x+2)$	A1
	So, $\dfrac{9x^2+20x-10}{(x+2)(3x-1)} \equiv 3+\dfrac{2}{(x+2)}-\dfrac{1}{(3x-1)}$	**(4)**
		(4 marks)

PJ@SELEXIQ

A level Mathematics Practice Paper – Algebra and functions – Mark scheme

	Source paper	Question number	New spec references	Question description	New AOs
1	C3 2013	1	2.6	Algebraic fractions	1.1b
2	C3 2017	1	2.6	Algebraic fractions	1.1b
3	C3 June 2014R	1	2.6	Simplification of rational expressions	1.1b
4	C3 2012	1	2.6	Algebra and functions	1.1b
5	C3 2016	1	2.6, 2.8	Composition of function	1.1b, 2.2a
6	C3 2013	2	2.7, 2.9	Modulus function, transformations	1.1b
7	C3 2017	3	2.3, 2.8	Functions, Inverses, Range	1.1b, 2.2a
8	C3 Jan 2012	2	2.9	Algebra and functions	1.1b
9	C3 Jan 2013	3	2.8, 2.9	Algebra and functions	1.1b, 2.2a
10	C3 June 2014	4	2.7, 2.9	Transforming graphs, modulus	1.1b, 2.2a
11	C3 2012	4	2.9	Algebra and functions	1.1b
12	C3 June 2014R	5	2.7,	Modulus function, Linear inequalities	1.1b, 3.1a
13	C3 June 2014	5	2.6, 2.8	Algebraic fractions, function work	1.1b, 3.1a
14	C3 2017	6	2.7, 2.9	Modulus graph, transformation and equation	1.1b, 2.2a, 3.1a
15	C4 2011	1	2.10	Partial fractions	1.1b
16	C4 Jan 2013	3	2.10	Partial fractions	1.1b

PJ@